LPS 105/205 C: Undecidability and Incompleteness:

Tuesday/Thursday 12:30-1:50 PM SSL 171

God exists because mathematics is consistent, and the devil exists because we cannot prove it.

-Andre Weil

Instructor:
Jeff Schatz (schatzj@uci.edu)
Office SST 795 – Office hours: Tuesday 3:30-4:30 PM, Wednesday 1:00-2:00 PM and Thursday 11:00-12:00 AM. Also, by appointment.

Course Description:
In the late 1920's and the 1930's, several of the most beautiful and far reaching theorems in the history of logic were developed in a surprisingly short period of time: these include the celebrated theorems of Church, Turing, Kleene, Tarski, and, especially, Gödel's incompleteness theorems. These theorems drastically altered the course of mathematical and philosophical logic in the 20th century, and remain highly influential in philosophy, mathematics, and computer science today; furthermore, they are frequently cited as exemplars of mathematical depth and beauty. The primary aim of this course will be to develop the necessary technical apparatus to state these theorems, and then prove the most significant of these, ending with philosophical reflections on their significance for mathematics and beyond. In so doing, we will have an opportunity to put many of the skills developed in Set Theory and Metalogic to use in pursuing a concrete and significant goal.

The course can roughly be thought of as split into two parts: in the first part (undecidability), we will introduce basic computability theory, developing the theory of Turing Machines, proving the existence of non-computable sets, and (briefly) introducing both degree theory and complexity theory. Our main theorems in this part of the class will be the Church-Turing(-Kleene) Thesis and Turing's Undecidability Theorem. While much of this material will be used in the second part of the course, we will study computability theory in its own right, going somewhat beyond what is required for the later theorems. Then, in the second part (incompleteness), we will combine these results from computability theory with a potpourri of metalogical results to prove Gödel's first and second incompleteness theorems. We will then conclude by examining the philosophical significance of these theorems for our understanding of mathematics, the relation of the mind to the brain, and the limits of human knowledge.

Quick Note on Terminology:
A very natural and common confusion sometimes results from the transition from 105/205 B to 105/205 C; in the former, we spend a quarter proving the completeness theorem, and in the latter, we spend a quarter proving the incompleteness theorems. How is this not flat-out contradictory?? We will consider this question briefly when the incompleteness theorems are introduced, but to make the matter somewhat clearer, we will use the following terminology: the completeness theorem of first-order logic was proved last quarter, and this quarter we will work to prove the incompleteness theorems **for theories in** first-order logic. If I ever speak of the incompleteness theorems, this full terminology is what is intended.

Required Texts:
You do not need to purchase any textbooks for this course, as all readings will be provided in digital format. Our main and overarching source will be a draft version of part III of Pen Maddy's textbook *Logic for Philosophers*. As this is a draft, DO NOT DISTRIBUTE IT TO ANYONE IN ANY WAY SHAPE OR FORM. EVER. Additionally, please let me know if you notice and typos or have any other suggestions; I will pass these along to Professor Maddy after the course is complete.

SERIOUSLY, DO NOT PUT THIS DRAFT ONLINE IN ANY FORM.

Beyond this, we will often use readings from the following sources, any of which would make a fine supplementary text for the different parts of the course:

Scott Soames, *Computability Theory*
Michael Sipser, Introduction to the Theory of Computation 3rd Edition
Herbert Enderton, A Mathematical Introduction to Logic, 2nd Edition
Wolfgang Rautenberg, A Concise Introduction to Mathematical Logic

Assignments/Evaluations:
The only assignments for this course will be take-home assignments: these include four homeworks (due weeks 3,5,7,9 at Friday at 2:30 PM) and one take-home final (due during finals week). You are welcome and encouraged to discuss the homeworks with your peers, though each must be completed and submitted individually. As a rough guideline, you are able to consult your peers and other textbooks while you are thinking about the problems, but when you are writing up your homework you should be alone with nothing but your notes and the official course readings in front of you. **If you have any concerns about what is an acceptable collaboration, please feel free to ask me.**

You are responsible for assuring that the homeworks are submitted correctly, either to the box outside of my office, or to my e-mail (schatzj@uci.edu). I encourage all students to submit homeworks digitally, as this will make submitting revisions much easier for you.

I will hand back homeworks at lecture on the Tuesday after they are due. You will have until the next Tuesday to submit revisions to these homeworks. If you lose X points on the homework, you can submit revisions for up to X/2 points (so if you get an 80% on the homework, you could attain a 90% with revisions). **You may not consult with your peers or look at their graded homeworks until your revisions are submitted.** Late homework can be submitted for restricted points: each business day that it is late, 10% will be subtracted from your score. **If you have not submitted the homework by the Tuesday after the due date, you will not be able to submit revisions**.

The final assignment will be a take-home final. **Unlike the homeworks, you are not permitted to discuss the problems on the final with any of your peers at all, and the only approved resources are your notes and the official course readings.** Any attempt to collaborate with other students or consult other resources would constitute plagiarism, and will be treated accordingly.

Grading Breakdown:
Officially, each homework and the final will be worth 20% of your final grade. However, if it benefits you mathematically, your lowest score homework will only count for 10%, and your final will count for 30% of your final grade. You do not need to contact me about which grading breakdown you would like to use; the most beneficial will be automatically applied on a student-by-student basis.

Assorted Policies:
Attendance- Class attendance is strictly mandatory, though attendance will not be taken.

Office Hours Appointments- If you are able to attend the three scheduled office hours, please do so. If none of these times work for you, please send me an e-mail to schedule a separate time: I am happy to make appointments during normal business hours (10 AM-5 PM) on any days that I am free. **If you stop by my office with questions outside of normal office hours without an appointment, I will be unable to meet with you.**

Extra Credit- There will be a small amount of extra credit for submitting a Midterm evaluation, provided that at least five students submit evaluations. There will be no further opportunities for extra credit.

Rounding- If you are within a .5% of the next highest letter grade, I will round you up to that grade. If you are within 1% of the next highest letter grade and both regularly attended the seminar and submitted each of the homeworks on time, I will round you up to that grade. There will be no other rounding of grades.

Academic Dishonesty and Plagiarism- The penalty for **any** violation of academic integrity—including but not limited to plagiarism—will be failure for the course and a letter recording the violation sent to the Dean. Please acquaint yourself with the university's official policy here: https://aisc.uci.edu/students/academic-integrity/index.php. **If there are any questions about whether something is acceptable or not, please contact me.**

Disabilities and Special Circumstances- I will make every effort to accommodate special needs of students. If you need special accomodations, please contact myself and the DSC (949-824-7494) at your earliest convenience.

Syllabus Revisions- Induction suggests that this syllabus will be changed as the course progresses, especially the schedule below. If you miss class, it is your responsibility to check on announcements and changes made in your absence.

Course Outline:

Date	Topic
4/3	Informal Computation and Algorithms
4/5	Models of Computation: RF's and URM's
4/10	Models of Computation: Turing Machines
4/12	Church-Turing(-Kleene) Thesis
4/17	Universal Turing Machines
4/19	The Halting Problem and Turing's Theorem
4/24	Oracle Computation
4/26	The Turing Degrees
5/1	Basics of Complexity Theory
5/3	The Arithmetical Hierarchy
5/8	Representation Theorems
5/10	Gödel Coding and the Arithmetization of Syntax
5/15	Fixed Point Theorems and the Undefinability of Truth
5/17	The First Incompleteness Theorem
5/22	The Derivability Conditions
5/24	Proving \Sigma_0^1 Completeness
5/29	The Second Incompleteness Theorem
5/31	Generalizations and Further Treatments
6/5	Philosophical Reflections: Epistemology
6/7	Philosophical Reflections: Philosophy of Mind and Metaphysics